EXERCISES [MAI 3.1-3.4]

3D GEOMETRY – TRIANGLES

SOLUTIONS

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A. Paper 1 questions (SHORT)

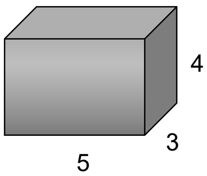
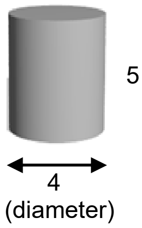
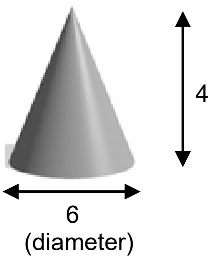
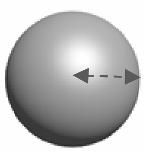
1. (a) $d_{AB} = \sqrt{3^2 + 4^2 + 0^2} = 5$

(b) $d_{OB} = \sqrt{1^2 + 1^2 + 5^2} = \sqrt{27} = 3\sqrt{3}$

(c) M(1/2,-1,5)

(d) C(-4,5,5)

2.

Solid	Volume	Surface area
	$V = 5 \times 3 \times 4 = 60$	$S = 2 \times 15 + 2 \times 12 + 2 \times 20 = 94$
	$r = 2$ $h = 5$ $V = \pi 2^2 5 = 20\pi$	$S = 2\pi(2)(5) + 2 \times (\pi 2^2) = 28\pi$
	$r = 3$ $h = 4$ $V = \frac{1}{3}\pi 3^2 4 = 12\pi$	slant height: $l = 5$ $S = \pi(3)(5) + \pi 3^2 = 24\pi$
 radius = 3	$V = \frac{4}{3}\pi 3^3 = 36\pi$	$S = 4\pi 3^2 = 36\pi$

3. (a) $V = \frac{1}{3}8^2 3 = 64$

(b) $AM^2 = 4^2 + 3^2 \Rightarrow AM = 5$

$S = 8^2 + 4 \times \left(\frac{1}{2} \times 8 \times 5\right) = 64 + 80 = 144$

4. (a) (i) $\sin B = \frac{4}{5} \Rightarrow B = 53.1$, (ii) $\cos B = \frac{3}{5} \Rightarrow B = 53.1$, (iii) (i) $\tan B = \frac{4}{3} \Rightarrow B = 53.1$

(b) $C = 180 - 90 - 53.1 = 36.9$

(c) (d) just confirm

(e) $A = \frac{1}{2}(3)(4) = 6$ $A = \frac{1}{2}(3)(5)\sin 53.1 \cong 6$ $A = \frac{1}{2}(4)(5)\sin 36.9 \cong 6$

5. (a) (i) $A = \frac{1}{2}(7)(5)\sin 40 \cong 11.2$ (ii) $BC^2 = 5^2 + 7^2 - 2(5)(7)\cos 40 \Rightarrow BC = 4.51$

(iii) $\frac{\sin 40}{4.51} = \frac{\sin B}{5} \Rightarrow B = 45.4$ and so $C = 180 - 40 - 45.4 = 94.6$

(b) $B = 27.3$, $A = 112.7$

(c) $C = 64.1$, $A = 75.9$ or $C = 115.9$, $A = 24.1$

6. (a) cosine rule: $7^2 + 9^2 - 2(7)(9)\cos 120° \Rightarrow$ AC =13.9 (= $\sqrt{193}$)

(b) **METHOD 1**

sine rule. $\frac{\sin \hat{A}}{9} = \frac{\sin 120}{13.9} \Rightarrow \hat{A} = 34.1°$

METHOD 2

cosine rule: . $\cos \hat{A} = \frac{7^2 + 13.9^2 - 9^2}{2(7)(13.9)} \Rightarrow \hat{A} = 34.1°$

7. the largest angle is opposite 7: $\cos \alpha = \frac{4^2 + 5^2 - 7^2}{2 \times 4 \times 5} = -\frac{1}{5} \Rightarrow \alpha = 101.5°$

8. (a) The smallest angle is opposite the smallest side.

$\cos \theta = \frac{8^2 + 7^2 - 5^2}{2 \times 8 \times 7} = \frac{88}{112} = \frac{11}{14} \Rightarrow \theta = 38.2°$

(b) Area = $\frac{1}{2} \times 8 \times 7 \times \sin 38.2° = 17.3\ \text{cm}^2$

9. (a) cosine rule: $4^2 + 6^2 - 2 \times 4 \times 6 \cos Q \Rightarrow P\hat{Q}R = 55.8°$

(b) Area = $\frac{1}{2} \times 4 \times 6 \sin 55.8 = 9.92\ (\text{cm}^2)$

10. (a) Angle $A = 80^\circ$

$\frac{\text{AB}}{\sin 40^\circ} = \frac{5}{\sin 80^\circ} \Rightarrow \text{AB} = 3.26\ \text{cm}$

(b) Area $= \frac{1}{2} ac \sin B = \frac{1}{2}(5)(3.26)\sin 60^\circ = 7.07\ \text{cm}^2$

11. (a) sine rule, $\frac{\sin R}{7} = \frac{\sin 75°}{10} \Rightarrow P\hat{R}Q = 42.5°$

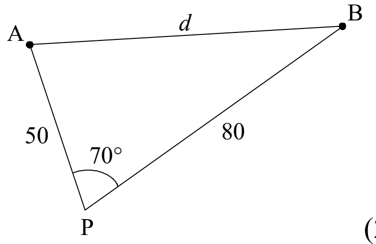
(b) $P = 180 - 75 - R = 62.5$

area Δ PQR = $\frac{1}{2} \times 7 \times 10 \times \sin 62.5 = 31.0\ (\text{cm}^2)$

12. Using sine rule: $\frac{\sin B}{5} = \frac{\sin 48°}{7} \Rightarrow \sin B = \frac{5}{7}\sin 48° \Rightarrow B = 32.06° = 32°$ (nearest degree)

13. $\cos C\hat{A}B = \dfrac{48^2 + 32^2 - 56^2}{2(48)(32)} \Rightarrow C\hat{A}B \approx 86°$

14.

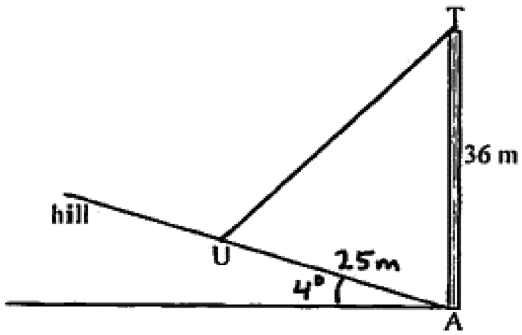


(2.5 × 20 = 50) (2.5 × 32 = 80)

$d^2 = 50^2 + 80^2 - 2 \times 50 \times 80 \times \cos 70° \Rightarrow d = 78.5$ km

15. Using area of a triangle: $20 = \frac{1}{2}(10)(8)\sin \boldsymbol{Q} \Rightarrow \sin Q = 0.5 \Rightarrow P\hat{Q}R = 30°$

16. (a)



(b) $T\hat{A}U = 86°$

cosine rule: $x^2 = 25^2 + 36^2 - 2(25)(36)\cos 86° \Rightarrow x = 42.4$

(c) $\frac{42.4}{\sin 86} = \frac{25}{\sin T} \Rightarrow T = 36.028 = 36°$

17. (a) cosine rule: $\cos A\hat{C}B = \dfrac{7^2 + 7^2 - 13^2}{2 \times 7 \times 7} \Rightarrow A\hat{C}B = 136°$

(b) **METHOD 1**

$A\hat{C}D = 180 - 136.4 = 43.6$

sine rule in triangle ACD: $A\hat{D}C = 47.9°$

$C\hat{A}D = 180 - (43.6... + 47.9...) = 88.5°$

METHOD 2

$A\hat{B}C = \frac{1}{2}(\pi - 2.381)\ \left(\frac{1}{2}(180 - 136.4)\right)$

sine rule in triangle ABD gives $A\hat{D}C = 47.9...°$

$C\hat{A}D = 180 - (43.6... + 47.9...) = 88.5°$

18. (a) $\dfrac{PQ}{40} = \tan 36° \Rightarrow PQ \approx 29.1$ m (3 sf)

(b) $A\hat{Q}B = 80°$

$\dfrac{AB}{\sin 80°} = \dfrac{40}{\sin 70°} \Rightarrow AB = 41\ 9.$ m (3 sf)

19. (a) $A\hat{B}C = 110°$

cosine rule: $AC^2 = 25^2 + 40^2 - 2(25)(40)\cos 110° \Rightarrow AC = 53.9$ (km)

(b) either by sine rule or by cosine rule, $B\hat{A}C = 44.2°$

bearing = 074°

B. Paper 2 questions (LONG)

20. (a) cosine rule $(AD)^2 = 7.1^2 + 9.2^2 - 2(7.1)(9.2)\cos 60° \Rightarrow AD = 8.35$ (cm)

(b) $180° - 162° = 18°$

sine rule: $\dfrac{DE}{\sin 18°} = \dfrac{8.35}{\sin 110°} \Rightarrow DE = 2.75$ (cm)

(c) $5.68 = \dfrac{1}{2}(3.2)(7.1)\sin D\hat{B}C \Rightarrow \sin D\hat{B}C = 0.5$

$D\hat{B}C$ 30° and/or 150°

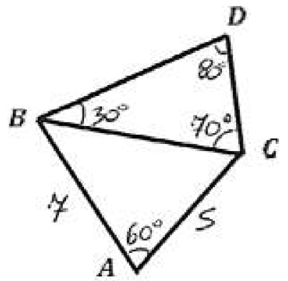
(d) $A\hat{B}C = (60° + D\hat{B}C) = 90°$

$(AC)^2 = 9.2^2 + 3.2^2 \Rightarrow AC = 9.74$ (cm)

(e) area of triangle ABD = $\dfrac{1}{2} \times 9.2 \times 7.1 \sin 60° = 28.28...$

Area of ABCD = 28.28... + 5.68 = 34.0 (cm^2)

21.



(a) $BC^2 = 5^2 + 7^2 - 2\cdot5\cdot7\cos 60 \Rightarrow BC = \sqrt{39}$

$\dfrac{BD}{\sin 70} = \dfrac{\sqrt{39}}{\sin 80} \Rightarrow BD = \sqrt{39}\,\dfrac{\sin 70}{\sin 80} \Rightarrow \boxed{BD = 5.96}$

(b) $A = A_{ABC} + A_{BDC} = \frac{1}{2}\,5\cdot7\sin 60° + \frac{1}{2}(5.96)\sqrt{39}\sin 30°$

$\Rightarrow A = 15.16 + 9.31 \Rightarrow \boxed{A \cong 24.5}$

(c) We need DC: $\dfrac{DC}{\sin 30°} = \dfrac{\sqrt{39}}{\sin 80°} \Rightarrow DC \cong 3.17$

Perimeter $\cong 5 + 7 + 5.96 + 3.17 \cong \boxed{21.1}$

(d) We need $A\hat{B}C$

$\dfrac{5}{\sin A\hat{B}C} = \dfrac{\sqrt{39}}{\sin 60} \Rightarrow \sin A\hat{B}C = \dfrac{5\sin 60}{\sqrt{39}} = 0.693$

$\Rightarrow A\hat{B}C = 43.9°$

Bearing $BA = 70° + 30° + 43.9° = \boxed{143.9°}$

22. (a) $BD = \sqrt{4^2 + 8^2 - 2\times 4\times 8\cos\theta}$

$BD = \sqrt{16(5-4\cos\theta)} = 4\sqrt{5-4\cos\theta}$

(b) (i) BD = 5.5653 ...

$\frac{\sin C\hat{B}D}{12} = \frac{\sin 25}{5.5653}$

$\sin C\hat{B}D = 0.911$

(ii) $C\hat{B}D = 65.7°$

$B\hat{D}C = 89.3°$

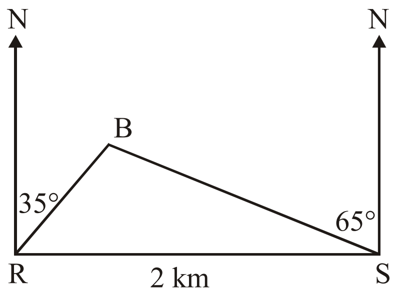
$\frac{BC}{\sin 89.3} = \frac{5.5653}{\sin 25}$ or $\frac{BC}{\sin 89.3} = \frac{12}{\sin 65.7}$ (or cosine rule)

BC = 13.2

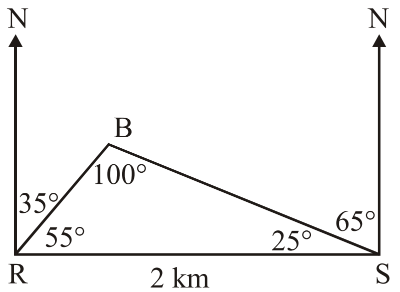
Perimeter = 4 + 8 + 12 + 13.2 = 37.2

(c) Area = $\frac{1}{2} \times 4 \times 8 \times \sin 40° = 10.3$

23. (a)



(b)

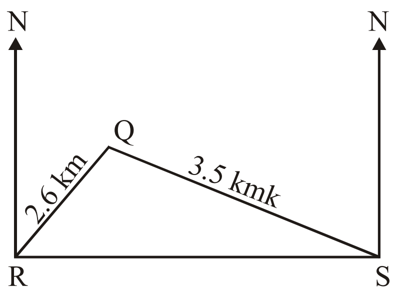


(i) $R\hat{B}S = 100°$

(ii) $R\hat{S}B = 25°$

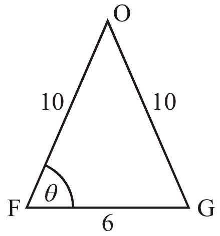
Using sine rule, $\frac{RB}{\sin 25°} = \frac{2}{\sin 100°} \Rightarrow RB = 0.858 = 0.9$ km or 900 m

(c)



$2^2 = 2.6^2 + 3.5^2 - 2(2.6)(3.5)\cos Q \Rightarrow \hat{Q} = 34.4° = 34°$ (to nearest degree)

24.



(a) (i) $\cos\theta = 3/10$ or cosine rule for $O\hat{F}G \Rightarrow \theta = 72.5°$ (3 s.f.)

(ii) $\tan\theta = h/3 \Rightarrow h = 3\tan\theta = 9.53939... = 9.54$ m (3 s.f.)

(iii) Area of $\Delta OFG = \frac{1}{2}(10)(6)(\sin\theta) = 30\sin\theta)$

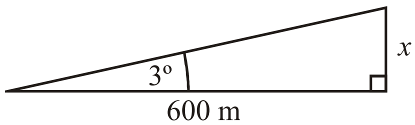
total surface area of roof $= 4 \times 30\sin\theta) = 114.4727... = 114$ m^2 (3 s.f.)

(iv) $\cos\varphi = 3/h \Rightarrow \varphi = 71.7°$ (3 s.f.)

(v) H = Height of tower from base to O $= 40 + \sqrt{h^2 - 3^2} = 49.055385... = 49.1$ m

(b) Height (BP) $= \dfrac{6\sin 79°}{\sin(90° - 79°)} = 30.9$ m

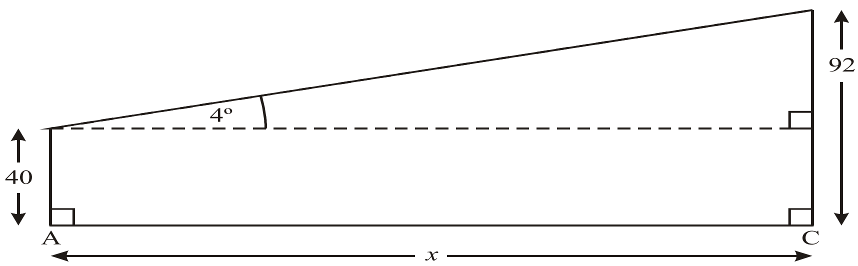
25. (a)



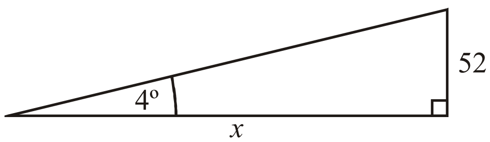
$\tan 3° = \dfrac{x}{600} \Rightarrow x = 600\tan 3° = 31.4447 = 31.4$ m

Therefore, height = 40 m + 31.4 m = 71.4 m

(b) (i)

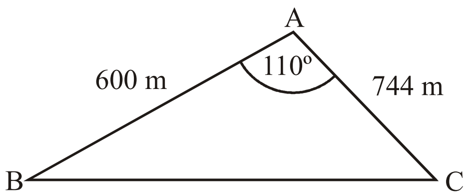


(ii)



$\tan 4° = \dfrac{52}{x} \Rightarrow x = \dfrac{52}{\tan 4°} = 743.6346453 = 744$ m

(c) (i)



$BC^2 = 600^2 + 744^2 - 2 \times 600 \times 744\cos 110° \Rightarrow BC = 1104$

(ii) $\dfrac{\sin c}{600} = \dfrac{\sin 110°}{1104} \Rightarrow \sin c = \dfrac{600 \times \sin 110°}{1104} \Rightarrow c = 30.710635° = 30.7°$

(iii) area $= \dfrac{1}{2} \times 600 \times 744\sin 110° = 209739.393 = 210000$ m^2 (3 s.f.)

26. (a) Sine rule $\frac{PR}{\sin 35} = \frac{9}{\sin 120} \Rightarrow PR = \frac{9\sin 35}{\sin 120} = 5.96$ km

(b) **EITHER** Sine rule $PQ = \frac{9\sin 25}{\sin 120} = 4.39$ km

OR Cosine rule: $PQ^2 = 5.96^2 + 9^2 - (2)(5.96)(9)\cos 25 = 19.29 \Rightarrow PQ = 4.39$ km

Time for Tom $= \frac{4.39}{8}$ Time for Alan $= \frac{5.96}{a}$

$\frac{4.39}{8} = \frac{5.96}{a} \Rightarrow a = 10.9$

(c) $RS^2 = 4QS^2 \Rightarrow 4QS^2 = QS^2 + 81 - 18 \times QS \times \cos 35 \Rightarrow QS = -8.20$ **or** $QS = 3.29$
therefore QS = 3.29

OR $\frac{QS}{\sin S\hat{R}Q} = \frac{2QS}{\sin 35} \Rightarrow \sin S\hat{R}Q = \frac{1}{2}\sin 35 \Rightarrow S\hat{R}Q = 16.7°$

$Q\hat{S}R = 180 - (35 + 16.7) = 128.3°$

$\frac{9}{\sin 128.3} = \frac{QS}{\sin 16.7} \Rightarrow QS = 3.29$

27. (a) $AC^2 = 5^2 + 4^2 - 2 \times 4 \times 5\cos x$
$AC = \sqrt{41 - 40\cos x}$

(b) $\frac{AC}{\sin x} = \frac{4}{\sin 30}, \Rightarrow \frac{1}{2}AC = 4\sin x \Rightarrow AC = 8\sin x$

(c) (i) $8\sin x = \sqrt{41 - 40\cos x}$
$x = 8.682...$, or $x = 111.317...$ hence $x = 111.32$ to 2 dp (obtuse)

(ii) $AC = 8\sin 111.32 = 7.45$

(d) (i) $7.45^2 = 32 - 32\cos y \Rightarrow \cos y = -0.734... \Rightarrow y = 137$

(ii) Area $= \frac{1}{2} \times 4 \times 4 \times \sin 137 = 5.42$

28. (a) (i) $AP = \sqrt{(x-8)^2 + (10-6)^2} = \sqrt{x^2 - 16x + 80}$

(ii) $OP = \sqrt{(x-0)^2 + (10-0)^2} = \sqrt{x^2 + 100}$

(b) $\cos O\hat{P}A = \frac{AP^2 + OP^2 - OA^2}{2AP \times OP} = \frac{(x^2 - 16x + 80) + (x^2 + 100) - (8^2 + 6^2)}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}}$

$= \frac{2x^2 - 16x + 80}{2\sqrt{x^2 - 16x + 80}\sqrt{x^2 + 100}} = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}}$

(c) For $x = 8$, $\cos O\hat{P}A = 0.780869 \Rightarrow O\hat{P}A = 38.7°$ (3 sf)

OR $\tan O\hat{P}A = \frac{8}{10} \Rightarrow O\hat{P}A = 38.7°$ (3 sf)

(d) $O\hat{P}A = 60° \Rightarrow 0.5 = \frac{x^2 - 8x + 40}{\sqrt{\{(x^2 - 16x + 80)(x^2 + 100)\}}} \Rightarrow x = 5.63$

(e) $O\hat{P}A = 0.$

(f) $x = \frac{40}{3}$ (= 13.333)

OR The line (OA) has equation $y = \frac{3x}{4}$. When $y = 10$, $x = \frac{40}{3}$

29. (a) $BC^2 = 65^2 + 104^2 - 2\,(65)\,(104)\cos 60° \Rightarrow BC = 91$ m

(b) area $= \frac{1}{2}(65)\,(104)\sin 60° = 1690\sqrt{3}$ (Accept $p = 1690$)

(c) (i) $A_1 = \left(\frac{1}{2}\right)(65)\,(x)\sin 30° = \frac{65x}{4}$

(ii) $A_2 = \left(\frac{1}{2}\right)(104)\,(x)\sin 30° = 26x$

(iii) $A_1 + A_2 = A \Rightarrow \frac{65x}{4} + 26x = 1690\sqrt{3} \Rightarrow \frac{169x}{4} = 1690\sqrt{3} \Rightarrow x = 40\sqrt{3}$